ERRATUM TO "THE FOURIER TRANSFORM FOR CERTAIN HYPERKÄHLER FOURFOLDS"

MINGMIN SHEN AND CHARLES VIAL

Let $X \subset \mathbb{P}^5$ be a smooth complex cubic fourfold. A line $l \subset X$ is called a *triple line* if there exists a plane $\Pi \subset \mathbb{P}^5$ such that $X \cap \Pi = 3l$.

The statement of [SV16, Proposition A.11] is erroneous ; we thank Moritz Hartlieb for bringing this to our attention. The following is a correction.

Proposition A.11. *Let $l \subset X$ be a line of second type. Assume that l is not contained in any plane in X and l is not a triple line. Then there is a unique line $l' \subset X$, disjoint with l, together with a natural isomorphism $\alpha_1 : \mathcal{E}_{[l]} \to l'$ and a degree 2 morphism $\alpha_2 : \mathcal{E}_{[l]} \to l$ such that the following holds : for all point $s \in \mathcal{E}_{[l]}$, the corresponding line $l_s \subset X$ is the line connecting the points $\alpha_1(s)$ and $\alpha_2(s)$. The surface S is smooth away from l.*

Proof. The proof is largely the same. Let $S = \mathbb{P}^3_{\langle l\rangle} \cap X$. Then S is a cubic surface singular along l. If S has a singular point $s_0 \in S$ which is not on l. Then the plane spanned by l and s_0 is contained in S. This contradicts the assumption that l is not contained in any plane in X. Thus S is smooth away from l. We pick homogeneous coordinates $[X_0 : X_1 : X_2 : X_3]$ on $\mathbb{P}^3_{\langle l\rangle}$ such that l is given by $X_2 = X_3 = 0$. Then $S \subset \mathbb{P}^3_{\langle l\rangle}$ is defined by an equation of the form

$$X_0Q_1(X_2, X_3) - X_1Q_0(X_2, X_3) - P(X_2, X_3) = 0$$

where Q_0 and Q_1 are homogeneous of degree 2 and P is homogeneous of degree 3.

Claim: The forms X_2Q_0, X_3Q_0, X_2Q_1 and X_3Q_1 are linearly independent.

By this claim, we conclude that there exist linear forms $L_0(X_2, X_3)$ and $L_1(X_2, X_3)$ such that

$$L_0Q_1 - L_1Q_2 + P = 0.$$

Then the equation for S can be written as

$$(X_0 + L_0)Q_1 - (X_1 + L_1)Q_0 = 0.$$

By the linear change of coordinates $X_0' = X_0 + L_0(X_2, X_3)$, $X_1' = X_1 + L_1(X_2, X_3)$, $X_2' = X_2$ and $X_3' = X_3$, the equation for S becomes

$$X_0'Q_1(X_2', X_3') - X_1'Q_0(X_2', X_3') = 0$$

one may conclude as in the proof of [SV16, Proposition A.11]. □

Proof of Claim. Firstly, we show that Q_0 and Q_1 are linearly independent. If Q_0 and Q_1 are proportional to each other, then after a linear change of coordinates the equation of S can be written in the form

$$X_0X_2X_3 - P(X_2, X_3) = 0 \qquad \text{or} \qquad X_0X_2^2 - P(X_2, X_3) = 0.$$

In either case, S is a cone over a singular cubic curve $C \subset (X_1 = 0)$ with vertex $v = [0 : 1 : 0 : 0]$. The curve C is irreducible since otherwise C contains a line component and hence S contains a plane which further contains l. Thus C has a unique singular point p and l is the line through v

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and p. There is a line $l_0 \subset (X_1 = 0)$ that intersects C only in the point p. Let Π be the plane spanned by l and l_0. Then Π intersects X only along l. Hence l is a triple line.

If X_2Q_0, X_3Q_0, X_2Q_1 and X_3Q_1 are not linearly independent, then there exist non-proportional linear forms $L_0(X_2, X_3)$ and $L_1(X_2, X_3)$ such that $L_0Q_0 = L_1Q_1$. Thus Q_0 and Q_1 share a common linear factor. Without loss of generality, we may and do assume that this common factor is X_2. Then the equation of S can be written as

$$X_2(X_0M_1 - X_1M_0) - P(X_2, X_3) = 0,$$

where M_0 and M_1 are non-proportional linear forms in X_2 and X_3. Note that P is not divisible by X_2 since otherwise l is contained in the plane $X_2 = 0$. We modify X_0 and X_1 by certain linear forms of X_2 and X_3 and obtain an equation of the above form with $P = X_3^3$. Then we again take $\Pi \subset \mathbb{P}^3_{\langle l\rangle}$ to be the plane defined by $X_2 = 0$. This plane Π meets X only along the line l. In other words, l is a triple line. This contradicts our assumptions. $\square$

In [SV16, Proposition A.12], the assumption that "l is not contained in any plane and l is not a triple line" is also needed.

Propositions A.11 and A.12 of [SV16] are only used in the proof of [SV16, Lemma 18.3]. The statement of [SV16, Lemma 18.3] is not affected. Indeed, it suffices to establish the statement of [SV16, Lemma 18.3] for a general point $s \in \Sigma_2$. Further, by specialization, if suffices to consider the case of a general cubic fourfold. However, a general cubic fourfold does not contain any plane, and, by [GK21, Thm. A], a general line of second type is not a triple line, so that the corrected versions of Proposition A.11 and Proposition A.12 apply.

References

[GK21] Frank Gounelas and Alexis Kouvidakis, *Geometry of lines on a cubic fourfold*, 2021.

[SV16] Mingmin Shen and Charles Vial, *The Fourier transform for certain hyperkähler fourfolds*, Mem. Amer. Math. Soc. **240** (2016), no. 1139, vii+163.

Universiteit van Amsterdam, Netherlands
E-mail address: m.shen@uva.nl

Universität Bielefeld, Germany
E-mail address: vial@math.uni-bielefeld.de